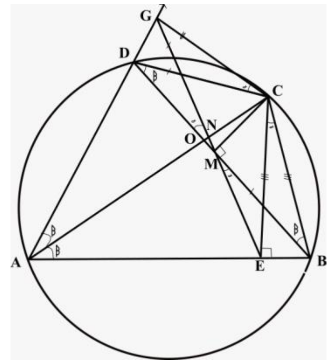
DIWALI BUMPER PRIZE – II PRIZE (Rs.3000) WINNER - Mr. Hara Gopal's Solution



Given :

AB > AO

BC=CD;

BM=MD & $\angle CEB = 90°$

TP:

AC = $\frac{EM \times MG}{CN}$

Solution:

In $\Delta\, BCD$; BC=CD and 'M' is the midpoint of BD ⇒ $\angle M = 90°$ [Isosceles Δ properties]

For ΔABD; C is a point on circumcircle and CE, CM are perpendicular to the 2 sides AB & BD then according to 'Simson line' property CG⊥ side AD ⇒ $\angle CGA = 90°$

Now, as quadrilateral CMEB is cyclic [∵ $\angle E = \angle M = 90°$]

$\angle BCE = \angle BME = \alpha$ [Same segment angles]

$\angle BME = \angle GMD = \alpha$ [V.O.A]

$\angle GMD = \angle GCD = \alpha$ [∵ Quadrilateral CMDG is also cyclic as $\angle M = \angle G = 90°$]

Now, consider ΔCBE & ΔCGD

$\angle E = \angle G = 90°$ [Proved]

$\angle CBE = \angle CDG$ [Exterior angle property of cyclic quadrilateral]

CB=CD (given)

∴ $\Delta\, CBE \cong \Delta CDG$ [AAS congruency]

∴ $CE = CG\ [CPCT]$ --------------------- (1)

In cyclic quadrilateral ABCD as BC = CD (given)

By isosceles triangle property and same segment angles property, we can prove that

$\angle CAB = \angle CAD = \beta$ [let us assume] ----------------- (2)

Now, In ΔCEA & ΔCGA

$\angle CAB = \angle CAD$ [from—(2)]

$\angle E = \angle G = 90$ [Proved)

CE=CG [from ----(1)

$\therefore \Delta CEA \cong \Delta CGA$ [AAS congruency]

$\therefore$ AE=AG [CPCT] ------------------- (3)

So quadrilateral AECG is a kite [$\because CG = CE$ & $AG = AE$]

and cyclic also [$\because \angle E = \angle G = 90$] sum = 180

Now, consider $\Delta\, ANE$ & ΔGNC

As $\angle CAE = \angle CGE$ &

$\angle GEA = \angle GCA$ [same segment angles are equal]

$\therefore \Delta$ ANE $\sim \Delta\, GNC$ [AA similarity]

$\frac{AN}{GN} = \frac{NE}{NC}$ [Proportionality of sides]

AN x NC = EN x NG

(AO+ON)CN = (EM+MN) (MG-MN)

AOxCN + ONxCN = EMxMG – EMxMN + MGxMN - MN^2

= EM x MG – EM x MN + (MN+NG) MN -MN^2

= EM x MG – EM x MN +MN^2 + NGxMN-MN^2

= EM x MC + MN (-EM+NG)

= EM x MG + MN (-EM+EN) [$\because$ NG=EN As AECG is a kite]

= EM x MG + MN (MN)

= EM x MG + MN^2

Here, In ΔCMO, $\angle M = 90°$ & $\angle N = 90°$ [As Diagonals meet at $90°$ in a kite]

So by Mean Proportional Theorem,

$$\boldsymbol{MN^2 = ON \times NC}$$

= EM x MG + ON x NC

$\therefore$AOxCN + ONxCN = EMxMG + ONxCN

$\therefore$ AOxCN =EMxMG

$\Rightarrow AO = \frac{EM \times MG}{CN}$ --------------------Hence Proved

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