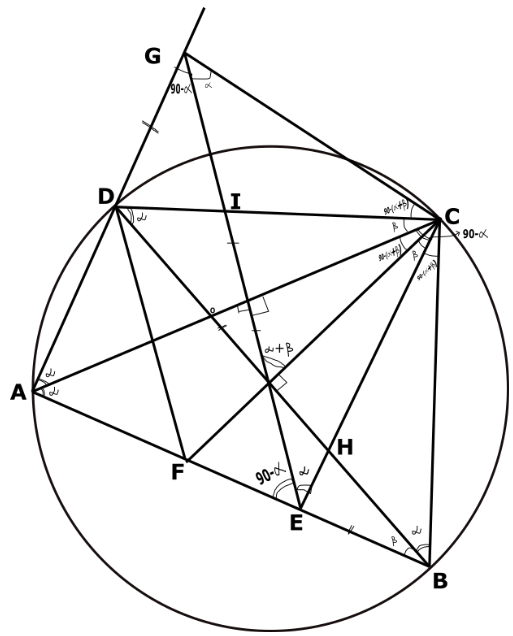
DIWALI BUMPER PRIZE Rs.5000 WINNER - Mrs. Madhumitha's Solution

Given:



ABCD is cyclic quadrilateral

CB = CD ⇒ BCD is isosceles triangle.

$\Delta CDB = \Delta CBD$ --------------------- (1)

As M is midpoint of D ⇒ CM Median and

$\angle CMB = 90°$ = $\angle CEB$ (given CE ⊥ AB)

⇒ EMCB is cyclic quadrilateral

As ABCD is cyclic quadrilateral

$\angle BAC = \angle BDC$ = $\angle DBC = \angle CAD = \alpha$ (let) (by (1)

⇒ AC is angle bisector of $\angle DAB$

$\frac{AD}{AE} = \frac{DO}{OB}$ -----------(2)

As EMCB is cyclic quadrilateral = $\angle MEC = \angle MBC = \alpha$

$\angle AEG = 180° - (90° - \alpha + 2\alpha)$ = $(90° - \alpha)$ ----------(a)

⇒ ΔAGE is isosceles ⇒ $AE = AG$ -------------(3)

$\frac{AG}{AE} = \frac{GN}{NE}$ ⇒ $\boldsymbol{N\ is}$ **midpoint GE**

⇒AN is both median & Altitude

⇒ $\angle ANG = 90°$

In Δ ACE, $\angle ACE$ = $90° - \alpha$ = $\angle AGE$ --- (by (a)

⇒ AECG is cyclic Quadrilateral

[AC& EG bisect at right angle and AG=AE ⇒ $AECG\ is$ kite ⇒ $CG = CE$]

In ΔBAD, EG is transversal

$\frac{BE}{EA} \times \frac{AG}{GD} \times \frac{DM}{MB} = 1$ (M is midpoint DM = MB) (ΔAGE is isosceles AG=AE)

⇒ $BE = GD$ -----------------------(4)

Also BC = CD (Given) & CE= CG (As $AECG$ is kite)

⇒ Δ $CGE \cong$ ΔCEB (By SSS Congruency)

⇒ $\angle AGC = 90°$

As AECG is cyclic quadrilateral by Ptolemy's Theorem

(AE$\times GC) + (AG \times EC)$ = AC$\times EG$ ----------------(5)

As AE=AG & GC=CE

2$(AE \times CE) = AC \times EG$ -------------------------- (6)

Also EN $\times NG = AN \times NC$

(EM+MN) (MG –MN) = (AN x NC)

(EM x MG) + (MN x MG) - (EM x MN) - MN^2 = AN x NC

(EM x MG) + MN x [MN+NG] – (EN-NM] x MN - MN^2=AN x NC

As EN = NG (As N is midpoint EG)

(EMx MG) + MN x (MN+NG) – (GN-NM) x MN - MN^2= AN x NC

EMx MG + MN^2+MN xNG – NG x MN + MN^2-MN^2= AN x NC

EM x MG + MN^2= AN x NC --------------------(7)

$$\frac{EM \times MG}{CN} = AN - \frac{MN^2}{CN}$$

T.P.T $AN - \frac{MN^2}{CN} = AO$

AO = AN – ON

=AN - $\sqrt{OM^2 - NM^2}$ (ΔOMN) ----------------- I

$\because \Delta NCM \sim \Delta MCO$ (by AA axiom)

$$\frac{NC}{MC} = \frac{CM}{CO} = \frac{NM}{MO} \Rightarrow OM = \frac{NM \times MC}{NC}$$

I-- becomes

AO =AN - $\sqrt{\frac{MN^2 \times MC^2}{NC^2} - MN^2}$

=AN - $\sqrt{\frac{MN^2[MC^2 - NC^2}{NC^2}}$

=AN - $\frac{MN}{NC}\sqrt{MC^2 - NC^2}$

=AN - $\frac{MN}{NC}\sqrt{MN^2}$

=AN - $\frac{MN^2}{NC}$ -------------------------- **Hence Proved**