__Solution__

Join CM & CG. For ΔABD, C is a point on its circumcircle.

∴ EMG is its Simpson line.

∴ CG⊥ AD

BC=CD (Given)

∴ $\angle BAC = \angle CAD = x$ (say)

∴ $\angle CBD = \angle CDB = x$

$\angle DCA = \angle DBA = z\ (say)$ (∵Angles in the same segment)

CB = CD and M is the midpoint of BD.

∴ $\angle CMB = 90°$

∴ $\angle CMB = \angle CEB = 90°$

∴ $BCME$ is concyclic. ∴ $\angle CEM = x$

Similarly ∠CMD + $\angle CGD = 180°$

∴ $CMDG$ is concyclic. ∴ $\angle CGM = x.$

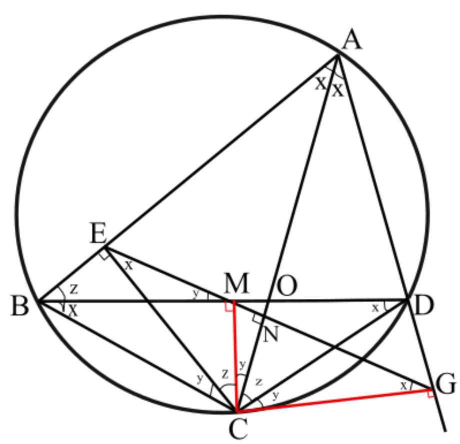
∴ $\angle BCE = \angle BME = y(say)$

∴ $\angle DMG = \angle DCG = y$

Now AC is angle bisector.

∴ The perpendicular lines from 'C' to AD & AB (ie CE & CG) are equal.

__Now see the picture__



AO x OC = BO x OD

AO = $\frac{BO\times OD}{OC}$ -------------------------(1)

$\Delta CNM \sim \Delta CGD$

$\frac{CN}{CG} = \frac{CM}{CD}$

$CN = \frac{CG\times CM}{CD}$ **----------------------- (2)**

$\Delta MCG \sim \Delta OCB$

$\frac{MG}{OB} = \frac{CG}{BC}$

MG = $\frac{OB\times CG}{BC}$ -------------------- (3)

$\Delta ECM \sim \Delta DCO$

$$\frac{CM}{CO} = \frac{EM}{DO}$$

$EM = \frac{CM \times DO}{CO}$ -------------------------------(4)

$\frac{(4)\times(3)}{(2)} \rightarrow \frac{EM \times MG}{CN} = \frac{CM \times DO}{CO} \times \frac{OB \times CG}{BC} \times \frac{CD}{CG \times CM}$ $(\because BC = CD)$

ie $\frac{EM \times MG}{CN} = \frac{OB \times OD}{OC}$ ---------------------- (5)

(1) +(5) $\rightarrow$

$AO = \frac{EM \times MG}{CN}$ ----------------------------Proved

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