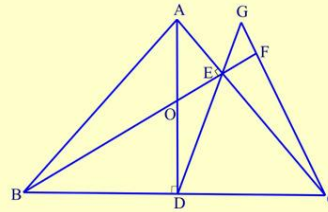


III CASH PRIZE WINNER Mrs.Ponmalar Selvi's Solution

In the adjoining picture,
AD & BE are altitudes of $\triangle ABC$
and O is the orthocentre.
F is a point on BE produced
such that $OE = EF$. DE &



CF meet at G. Prove: $\frac{BC}{GC} = \frac{FC}{DC}$.

Question created by
Dr. M. Raja Climax
Founder Chairman,
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Solution:

$$\angle OEC + \angle ODC = 180^\circ$$

$\therefore ODCE$ is concyclic

$$\text{Let } \angle ECD = \alpha$$

$$\angle BOD = \alpha \text{ (Exterior angle is equal to interior opposite angle)}$$

$$\angle AOE = \angle BOD = \alpha \text{ (vertically opposite angles)}$$

$$\angle DBO = \angle OAE = \beta \text{ (Say)}$$

$\therefore ABDE$ is concyclic

$$\text{Let } \angle ABE = \angle ADE = \gamma$$

$$\angle EDC = 90^\circ - \gamma$$

$CE \perp OF$ and bisect of

$\triangle OCF$ is an isosceles triangle

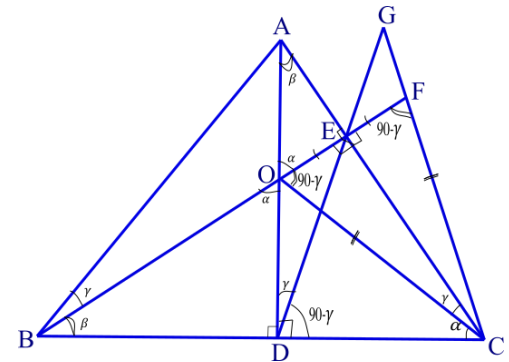
$$OC = CF$$

$$\angle COF = \angle CFO \text{ (Angles opposite to equal sides are equal)}$$

$$\angle ODE = \angle OCE = \gamma \text{ (Angles in the same segment)}$$

$$\angle COE = 90^\circ - \gamma$$

$$\angle CFE = \angle COE = 90^\circ - \gamma$$



Consider ΔBCF & ΔDCG $\angle C$ is common

and $\angle BFC = \angle CDG = 90^\circ - \gamma$

$\therefore \Delta BCF \sim \Delta DCG$ (by AA similarity)

\therefore *Corresponding sides are proportional.*

$$\frac{BC}{GC} = \frac{FC}{DC}$$
