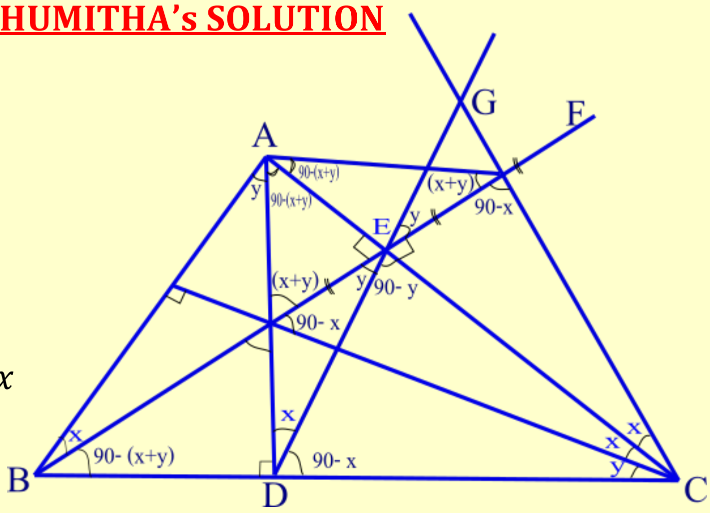
Cash Prize Winner Mrs.MADHUMITHA's SOLUTION



Given

ABC is a triangle. Join OC & AF

$\angle BDA = \angle BEA = 90°$

$\Rightarrow ABDE$ is cyclic

Let $\angle ABE = \angle ADE = x \Rightarrow \angle \text{EDC} = 90° - x$

$\angle BAD = \angle \text{BED} = \text{y}$

$\Rightarrow \angle EBD = \angle DAE = 90 - (x + y)$

As 'O' is orthocentre $\angle OCB = y$ [CO extended to meet AB at H CH is altitude]

And $\angle DCA = x + y$, $\angle OCA = x$

As OE = EF & AE is common

$\angle AEO = \angle AEF = 90°$

$\Delta AEO \cong \Delta AEF$ $(by\ SAS\ congruecny)$

$\Rightarrow \angle OAE = \angle EAF = 90 - (x + y)$

$\angle AOE = \angle AFE = x + y$

$\angle AOE = \angle AFE = x + y$

$\angle AFB = \angle AFE = x + y \Rightarrow$ ABCF is cyclic quadrilateral

similarly EC is common, EC = EF

$\angle OEC = \angle CEF = 90°$

$\Delta OEC \cong \Delta FEC$ (by SAS congruency)

$\angle OCA = x = \angle ECF$

consider ΔBFC & ΔDCG

$\angle BCF = \angle DCG$ & $(common)$

$\angle BFC = \angle GDC = 90 - x$

By AA axiom

$\Delta BFC \sim \Delta GDC$

$$\Rightarrow \frac{BF}{GD} = \frac{FC}{DC} = \frac{BC}{GC}$$