

I CASH PRIZE WINNER Master.H.Karthikeya's SOLUTION

Given:

$$OE=EF, \angle D=90^\circ=\angle E$$

T.P.:

$$\frac{BC}{GC} = \frac{FC}{DC}$$

Construction:

Draw CH perpendicular to AB.

Proof:

In $\triangle AOE$ & $\triangle BOD$,

$$\angle AOE = \angle BOD \text{ (Vertically Opposite Angles)}$$

$$\angle OED = \angle BDO = 90^\circ$$

$\triangle AOE \sim \triangle BOD$ (AA Similarity)

$$\text{So, } \angle A = \angle B \text{ -----(1)}$$

In $\triangle CEO$ & $\triangle CEF$,

$$\angle E = \angle E = 90^\circ$$

$CE=CE$ (Common side)

$OE=EF$ (Given)

$\triangle CEO \cong \triangle CEF$ (SAS similarity)

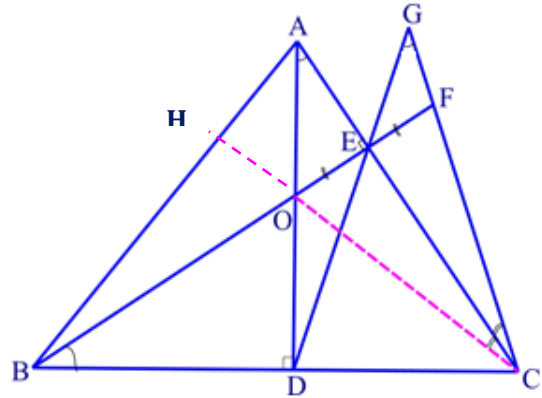
$$\text{So, } \angle OCE = \angle FCE \text{ ----- (2)}$$

In ODCE quadrilateral,

$$\text{As } \angle D + \angle E = 90^\circ + 90^\circ = 180^\circ$$

This is a cyclic quadrilateral.

$$\text{So, } \angle ECO = \angle EDO \text{ (Same segment angles) ----- (3)}$$



In $\triangle ADE$ & $\triangle ACO$,

$$\angle AED = \angle AOC \text{ ----- (4)}$$

$$\angle AED = \angle GEC \text{ (Vertically opposite angles)}$$

$$\angle AOC = \angle GEC \text{ -----(5)}$$

In $\triangle AOC$ & $\triangle GEC$

$$\text{As } \angle AOC = \angle GEC$$

$$\angle OCA = \angle ECG$$

$\triangle AOC \sim \triangle GEC$ (AA similarity)

$$\angle A = \angle G$$

$$\text{Hence, } \angle B = \angle G$$

In $\triangle BCF$ & $\triangle GCD$

$$\angle B = \angle G$$

$$\angle C = \angle C$$

$\triangle BCF \sim \triangle GCD$ (AA similarity)

$$\Rightarrow \frac{BC}{GC} = \frac{FC}{DC} \text{ ----- Hence proved.}$$