Author's Solution

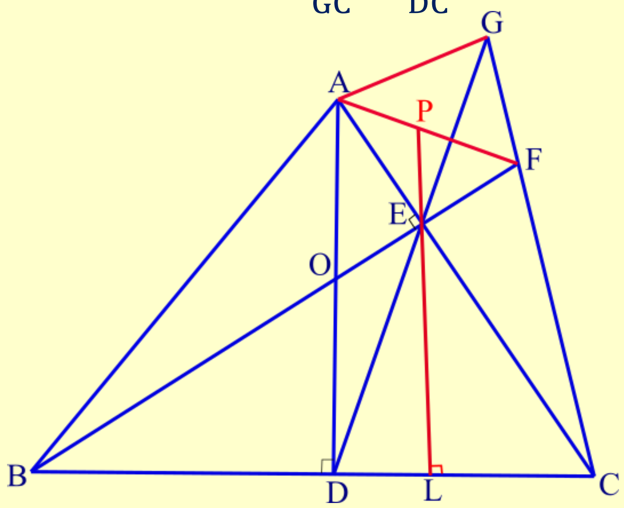
All the prize winners have solved the problem almost in a similar way. But, the author has, as usual, solved it in a different way. Yes, he displays his own distinct and peculiar style in coining out the solution. It is quite interesting to see how he solves the problem using just the concept of Brahmaguptha Theorem and Simpson line.

- Site Administrator

SOLUTION

Given :

InΔABC, AD & BE are altitudes and 'O' is the Orthocentre. 'F' is a point on BE produced such that OE=EF. DE & CF meet at G. To prove $\frac{\text{BC}}{\text{GC}} = \frac{\text{FC}}{\text{DC}}$.



Solution:

Construction :

Join AF & AG. Draw EL ⊥ BC

and produce LE to meet AF at P.

Proof:

PL ∥ AD and OE = EF

∴ AP = PF --------------- (1) [∵ In Δ AOF, PE ∥ AO]

Now, ABCF is a quadrilateral with diagonals BF & AC perpendicular to each other at E. The line perpendicular to side BC through E bisects the side AF.

∴ As per the converse of Brahmagupta's Theorem,

ABCF is concyclic.

∴ $\angle CAF = \angle CBF$ --------------------- (2)

Since ABCF is concyclic, A is a point on the circumcircle of ΔBCF and AD & AE are perpendiculars drawn from A to its sides BC & BF respectively and therefore, DEG is its Simpson line (Pedal line) and hence AG⊥ CF (Produced).

$\therefore AEFG$ is concyclic & $\angle EGF = \angle EAF$ ---------------------- (3)

(2) & (3) ⟶ $\angle DGF = \angle DBF$

$\Rightarrow GBDF$ is concyclic

$\Rightarrow GF$ & BD are chords intersecting outside the circle at C.

∴ BC x CD = GC x CF

$\Rightarrow \frac{BC}{GC} = \frac{CF}{CD}$ ------------------ Proved

Dr. M. Raja Climax
Founder Chairman,
CEOA Group of Institutions

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