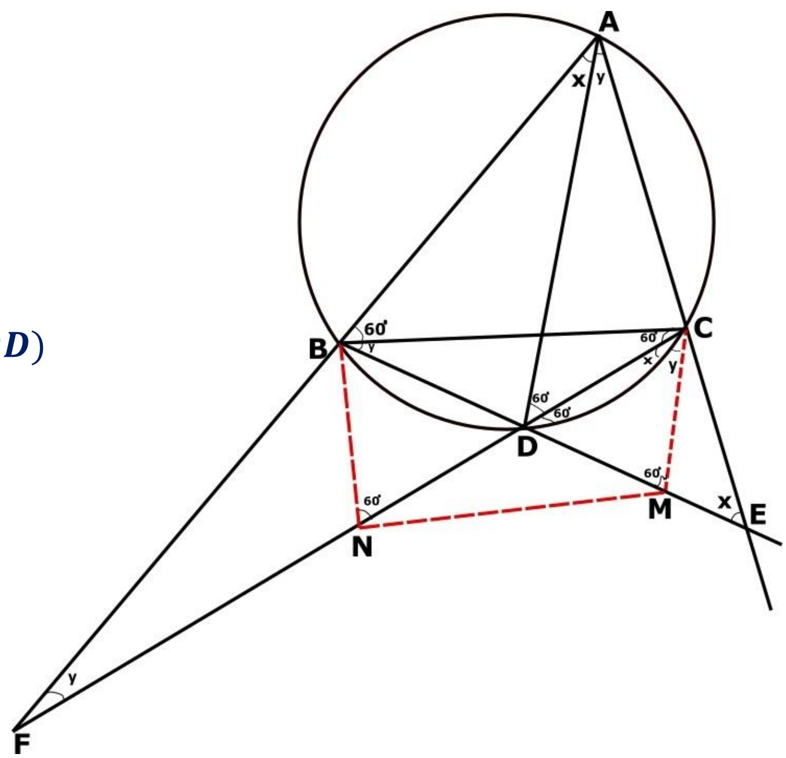
Corollaries:

1. $DC^2 = BD \times ME = BD(DE - OD)$
2. $AD \times BC = (BF \times CD) + (EC \times BD)$
3. $\frac{BD}{DF} + \frac{CD}{ED} = 1$
4. $\frac{1}{AE} + \frac{1}{AF} = \frac{1}{BC}$
5. $AD^2 = BC^2 + BD \times DC$

Corollary: 1



$\Delta BDC \sim \Delta CME$

$$\frac{BD}{CM} = \frac{DC}{ME}$$

$DC \times CM = BD \times ME$

$DC^2 = BD \times ME = BD(DE - OD)$

Corollary: 2

$\Delta BNF \sim \Delta CME$

$$\therefore \frac{BN}{CM} = \frac{NF}{ME} = \frac{BF}{CE}$$

$BN \times CE = CM \times BF$

$\Rightarrow BD \times CE = BF \times CD$ ------------------------(1)

$\Delta ADC \sim \Delta FDB$

$$\therefore \frac{AD}{FD} = \frac{DC}{DB} = \frac{AC}{BF}$$

$\Rightarrow AC \times BD = BF \times CD$ ----------------------- (2)

$\Delta ABD \sim \Delta ECD$

$\therefore \frac{AB}{EC} = \frac{BD}{CD} = \frac{AD}{ED}$

$AB \times CD = EC \times BD$ ------------------------------ (3)

(2) + (3) $\Rightarrow (AC \times BD) + (AB \times CD)$

= $(BF \times CD) + (EC \times BD)$

$\Rightarrow$ BC$(CD + BD) = (BF \times CD) + (EC \times BD)$

$\Rightarrow AD \times BC = (BF \times CD) + (EC \times BD)$

Corollary: 3

In the current month's problem, we have proved,

$AD^2 = (BD \times DE) + (CD \times DF)$ -----------------(1)

Now,

$\Delta ADF \sim \Delta EDA$

$\therefore \frac{AD}{ED} = \frac{DF}{DA}$

$\Rightarrow AD^2 = ED \times DF$ -------------------------- (2)

(1) & (2) $\Rightarrow$

$(BD \times DE) + (CD \times DF) = ED \times DF$

$\Rightarrow (BD \times DE) + (CD \times DF) = ED \times DF$

$\Rightarrow \frac{(BD \times DE) + (CD \times DF)}{ED \times DF} = 1$

$\Rightarrow \frac{BD}{DF} + \frac{CD}{ED} = 1$

Corollary: 4

We know from the previous corollary,

$$\frac{BD}{DF}+\frac{CD}{ED}$$ ---------------------- (1)

Now, $\Delta EBA \sim \Delta ECD$

$$\therefore \frac{EB}{EC}=\frac{AB}{CD}=\frac{EA}{ED}$$

$$\Rightarrow \frac{AB}{AE} = \frac{CD}{ED}$$ ---------------(2)

$$\Delta\, FAC \sim \Delta\, FDB$$

$$\therefore \frac{FA}{FD}=\frac{AC}{BD}=\frac{FC}{FB}$$

$$\Rightarrow \frac{AC}{AF}=\frac{BD}{DF}$$ --------------------------(3)

(2)& (3) $\Rightarrow$

$$\frac{AB}{AE}+\frac{AC}{AF}=\frac{CD}{ED}+\frac{BD}{DF}$$ ---------------------- (4)

(1) & (4) $\Rightarrow$

$$\frac{AB}{AE}+\frac{AC}{AF}=1$$

$$\Rightarrow BC\left[\frac{1}{AE}+\frac{1}{AF}\right]=1 \qquad (\because AB=AC=BC)$$

$$\Rightarrow \frac{1}{AE}+\frac{1}{AF}=\frac{1}{BC}$$

Corollary: 5

Construction:

Join MN. BN ∥ $CN = BD + CD = AD$ **----------------------(1)**

BNMC is concyclic & an isosceles trapezium

$\therefore$ A**s per Ptolemy's theorem,**

$BM \times CN = BC \times MN + BN \times CM$

$\Rightarrow \; AD^2 = \; BC^2 + BD \times DC$