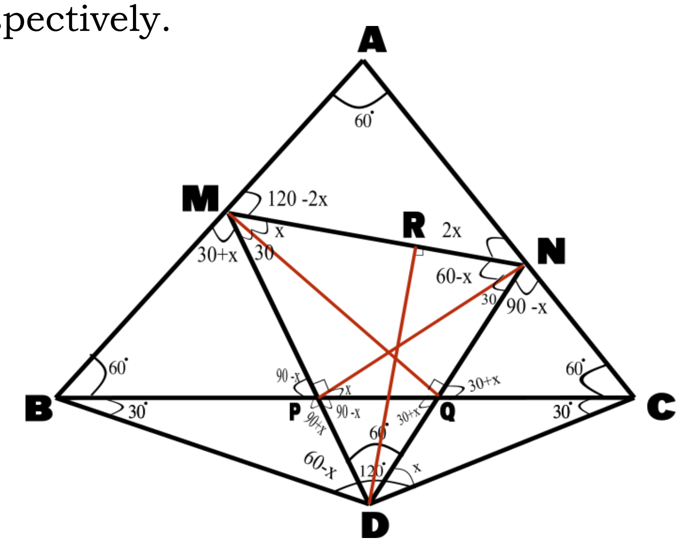
SOLUTION 2:

To prove:- MN+AN+AM = AB +AC

i.e., MN = MB +NC

Let DM and DN cut BC at P and Q, respectively.

Join MQ and NP.



Let $\angle CDN$ be x^o.

$$\angle ABD = \angle ACD = 90^o$$

$$\angle NCP = \angle NDP = 60^o$$

∴ NPDC is concyclic.

$$\angle MDQ = \angle MBQ = 60^o$$

∴ MBDQ is concyclic.

Now therefore, the measurements of all the angles are as shown in the picture.

In ΔMBD,

$$\frac{MB}{\sin(60-x)} = \frac{a}{\sin(30+x)}$$

$$MB = a\left[\frac{\sin(60-x)}{\sin(30+x)}\right] \ldots \ldots \ldots \ldots \ldots (1)$$

In ΔNDC,

$$\frac{NC}{\sin x} = \frac{a}{\sin(90-x)}$$

NC = $a \tan x$ …………………(2)

(1) + (2)→ MB +NC = $a\left[\frac{\sin(60-x)}{\sin(30+x)}\right] + a\tan x$

$= = a[\tan x + \tan(60 - x)]$ ---------- (3)

In $\Delta\, MBD$,

$$MD = \frac{a}{\sin(30+x)}$$

In $\Delta\, NDM$, $\frac{MD}{\sin(90-x)} = \frac{MN}{\sin 60}$

$$MN = MD \times \frac{\sin 60}{\cos x} = \frac{a(\sin 60)}{\sin(30+x)\cos x} \quad \ldots\ldots\ldots\ldots (4)$$

now we have to prove that

(3) & (4) are equal

TPT : $\tan x + \tan(60-x) = \frac{\sin 60}{\sin(30+x)\cos x}$

LHS : $\tan x + \tan(60-x)$

$$= \frac{\sin x}{\cos x} + \frac{\sin(60-x)}{\cos(60-x)}$$

$$= \frac{\sin x\cos(60-x)+\cos x\ \sin(60-x)}{\cos x\cos(60-x)}$$

$$= \frac{\sin(\cancel{x}+60-\cancel{x})}{\cos x\cos(60-x)}$$

$$= \frac{\sin 60}{\cos x\sin(90-60-x)} = \frac{\sin 60}{\cos x\sin(30+x)} = \text{RHS}$$