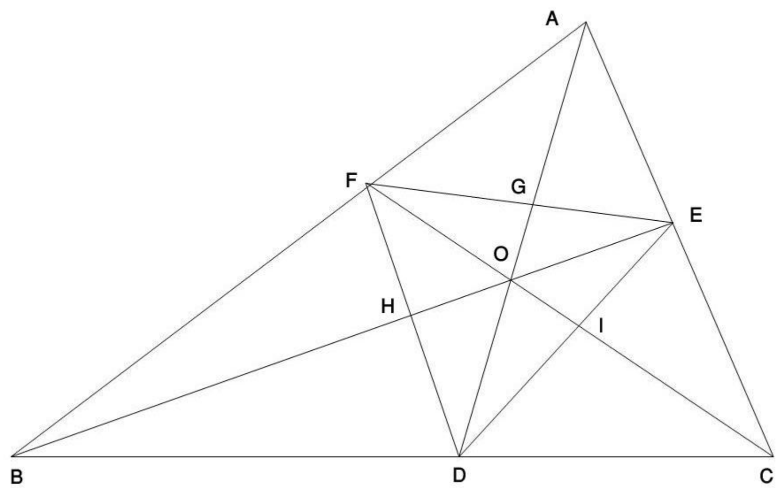
The Concurrency Theorem:



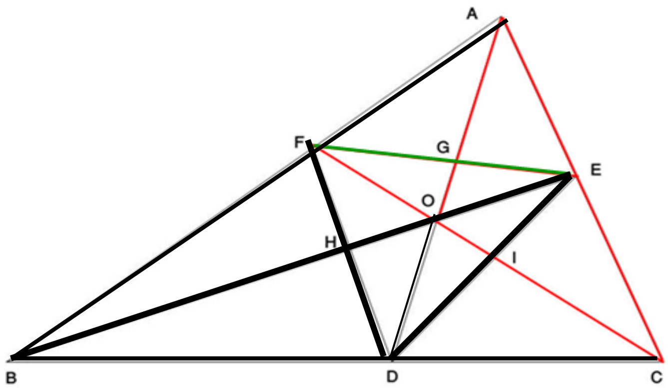
Given:- **In ΔABC, D, E and F are base points on sides BC, AC and AB respectively such that AD, BE and CF (Cevians) are concurrent at O. FE and AD intersect at G; FD and BE at H; and DE and CF at I.**

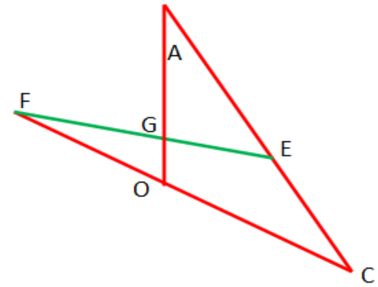
To prove:

$$\frac{AG}{GO} = \frac{AD}{DO} \; ; \; \frac{BH}{HO} = \frac{BE}{EO} \; ; \text{ and } \; \frac{CI}{IO} = \frac{CF}{FO}$$

Proof :

For Δ AOC, EGF is a transversal cutting the sides AC, AO and CO at E, G and F respectively.

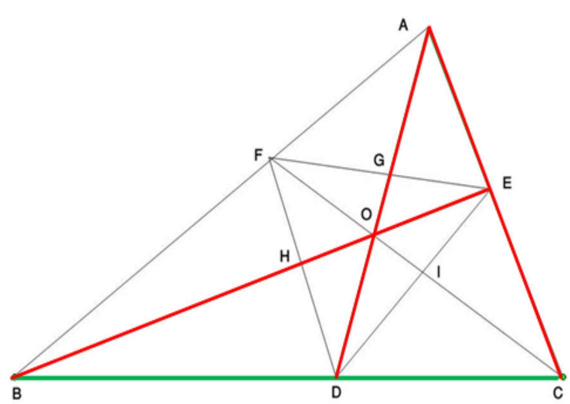


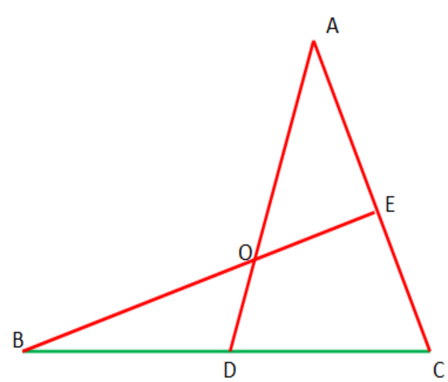


∴ as per Menelaus Theorem,

$$\frac{CE}{EA} \ X \ \frac{AG}{GO} \ X \ \frac{OF}{FC} = 1 \text{--} \quad (1)$$

For Δ AOE, CDB is a transversal cutting its sides AE, AO & OE at C, D and B respectively.





$\therefore$ As per Menelaus Theorem,

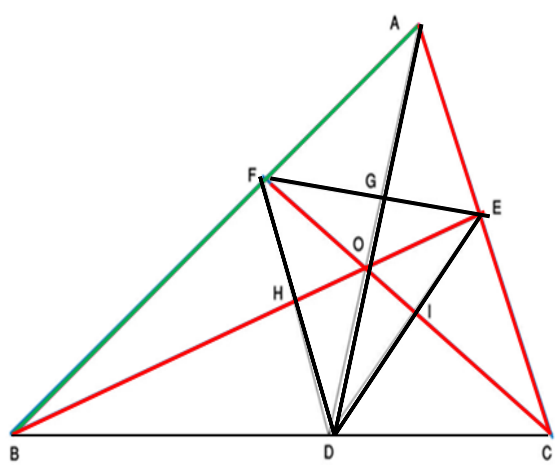
$$\frac{AC}{CE} \ X \ \frac{EB}{BO} \ X \ \frac{OD}{DA} = 1 \text{ -- } (2)$$

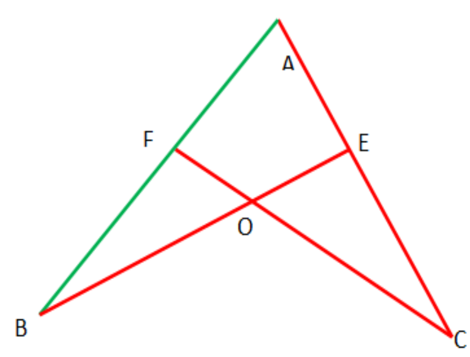
(1) x (2) $\longrightarrow$

$$\frac{\cancel{CE}}{EA} \times \frac{AG}{GO} \times \frac{OF}{FC} \times \frac{AC}{\cancel{CE}} \times \frac{EB}{BO} \times \frac{OD}{DA} = 1$$

ie $\left(\frac{AG}{GO} \times \frac{OD}{DA}\right) = \left(\frac{EA}{AC} \times \frac{CF}{FO} \times \frac{OB}{BE}\right)$ ---------------------------- (3)

For ΔCOE, BFA is a transversal cutting its sides EO, CO and CE at B, F & A respectively.





$\therefore$ As per Menelaus Theorem,

$$\frac{EA}{AC} \times \frac{CF}{FO} \times \frac{OB}{BE} = 1 \quad \text{--------------------------------} \quad (4)$$

(3) & (4) gives

$$\frac{AG}{GO} \times \frac{OD}{DA} = 1$$

ie $\frac{AG}{GO} = \frac{AD}{DO}$

Similarly,

We can prove

$\frac{BH}{HO} = \frac{BE}{EO}$ and

$\frac{CI}{IO} = \frac{CF}{FO}$

- 'CONCURRENCY THEOREM' proved.
