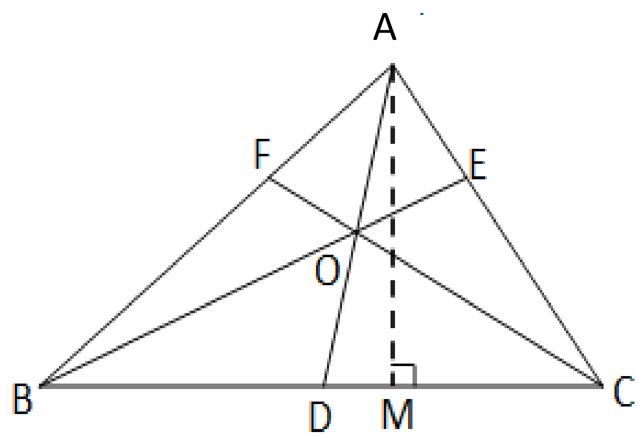
The Ceva's Theorem:

When three cevians of a triangle ABC namely AD, BE and CF are concurrent at O, then,

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$

The proof of this property is as follows:

Proof:



Draw AM $\perp$ BC

Δ ABD and Δ ACD are having common heights in the altitude AM drawn from A to the base BC.

The area of Δ ABD = ½ x AM x BD

The Area of Δ ACD = ½ x AM x DC

$$\therefore \frac{Area of\ \Delta\ ABD}{Area of\ \Delta\ ACD} = \frac{\frac{1}{2} \text{ x AM x BD}}{\frac{1}{2} \text{ x AM x DC}} = \frac{BD}{DC}$$

ie. $\frac{BD}{DC} = \frac{\Delta\ ABD}{\Delta\ ACD}$ [Where Δ denotes to the area of the triangle] ------------------- (1)

Similarly we can prove that $\frac{BD}{DC} = \frac{\Delta\, OBD}{\Delta\, ODC}$ [Where Δ denotes to the area of the triangle] ---------------------- (2)

(1) & (2) ⟶ $\frac{BD}{DC} = \frac{\Delta\, ABD - \Delta\, OBD}{\Delta\, ACD - \Delta\, ODC}$ [Each ratio = $\frac{difference \;\; between \;\; numerators}{difference \;\; between \;\; denominators}$

ie. $\frac{BD}{DC} = \frac{\Delta\, OAB}{\Delta\, OAC}$ -------------------------------------- (3)

Similarly, it can be proved that $\frac{CE}{EA} = \frac{\Delta\, OBC}{\Delta\, OAB}$ ------------------------------- (4)

And $\frac{AF}{FB} = \frac{\Delta\, OAC}{\Delta\, OBC}$ --------------------------------- (5)

(3), (4) & (5) ⟶ $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = \frac{\Delta\, \cancel{OAB}}{\cancel{\Delta\, OAC}} \times \frac{\Delta\, \cancel{OBC}}{\cancel{\Delta\, OAB}} \times \frac{\Delta\, \cancel{OAC}}{\cancel{\Delta\, OBC}} = 1$

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$ -------------------------- Proved