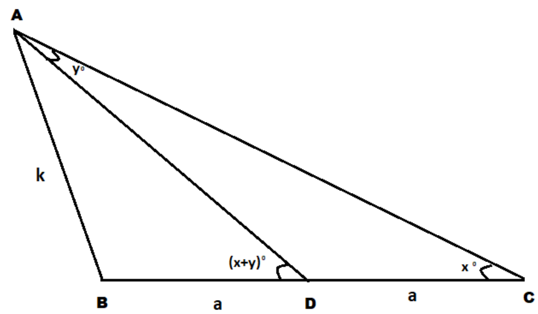
Solution:

Here Δ ABC is a special triangle with angles A, B &C being 45° , 105° & 30° respectively. Only when the angles are in this combination, does the median from A, create $\angle BAD$ as an equivalent of $\angle C$. Let us now study whether there are other triangles (with different combination of angles), where the median AD from A makes $\angle BAD$ equal to $\angle C$. Let us take the following picture.



In this picture, ABC is a Δ. AD is its median from A. Let $\angle C = \angle BAD = x°$.

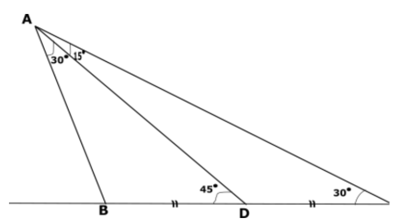
Let $\angle CAD = y°$

$\therefore \angle ADB = (x+y)°$

Let AB = k & BD & DC = a

Now Δ ABD, $\frac{k}{Sin(x+y)} = \frac{a}{Sin(x)}$

$k = a\left[\frac{Sin(x+y)}{Sinx}\right]$ --- (1)



In Δ ABC,

$$\frac{k}{Sinx} = \frac{2a}{Sin(x+y)}$$

ie $k = 2a\left[\frac{Sinx}{Sin(x+y)}\right]$ --- (2)

(1) & (2) → $a\left[\frac{Sin(x+y)}{Sinx}\right]$=$2a\left[\frac{Sinx}{Sin(x+y)}\right]$

ie $Sin^2(x+y) = 2Sin^2x$

ie $Sin(x+y) = \sqrt{2}Sin\,x$

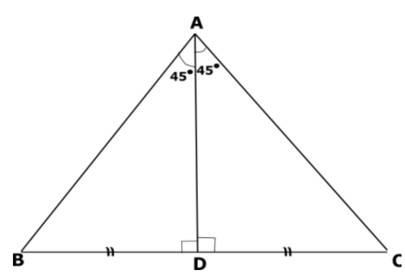
If $(x+y)$ = 45°,

Sin 45° =$\sqrt{2}Sin\,x$

$\therefore Sin\,x = \frac{1}{2}$

$x = 30°$

If $(x+y) = 90\,°$

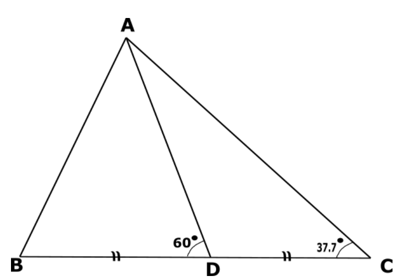


Then, Sin 90° = $\sqrt{2}Sin\,x$

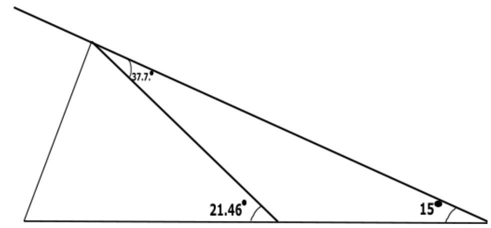
$Sin\,x = \frac{1}{\sqrt{2}}$

$x = 45°$

If $(x+y) = 60°$



Sin 60° = $\sqrt{2}Sin\ x$



$Sin\ x = \frac{\sqrt{3}}{2\sqrt{2}} = 0.61245$

$Sin^{-1}(0.61245) = 37.7°$

$If\ x = 15°$

$Sin(x + y) = \sqrt{2}Sin\ 15°$

$= \sqrt{2}\ (0.2588) = 0.3659$

$Sin^{-1}0.3659 = 21.46°$

$\therefore (x + y) = 21.46°$

Like the above ones,

We can find infinity number of Δs where the median AD gives $\angle BAD = \angle C$.

Dr. M. Raja Climax
Founder Chairman, CEOA

****