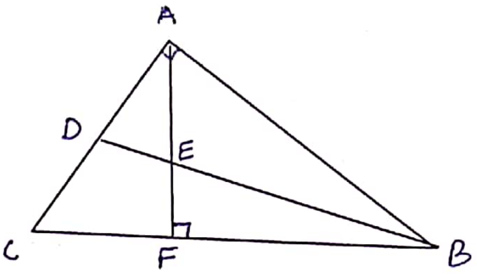
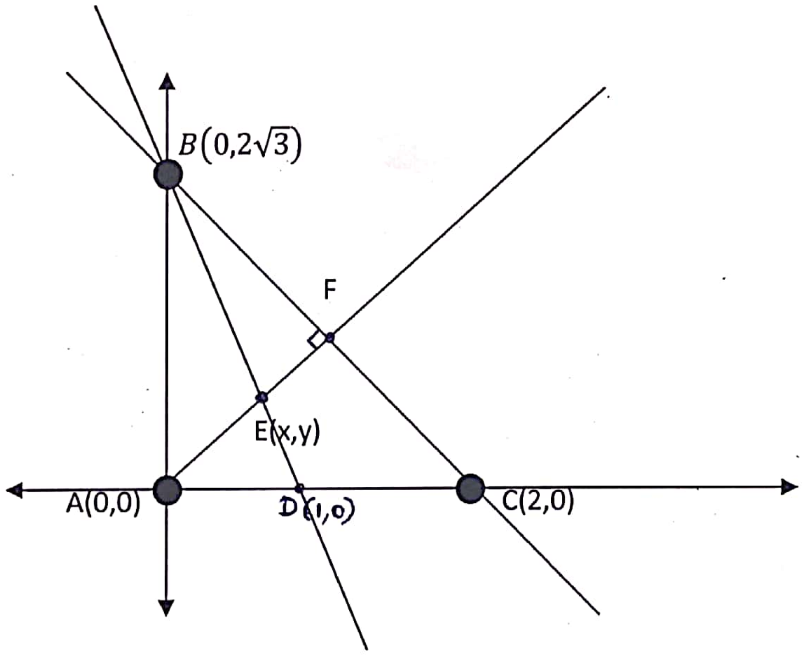
Problem:



In this figure, $\angle CAB = 90°$; $AF \perp CB$ and BD is Median. $AC = 2,\ AB = 2\sqrt{3}.$ Find out $AE.$

<u>Solution:</u>



In a Triangle ΔABC ,

Given $\angle CAB = 90,\ AC = 2,\ AB = 2\sqrt{3},\ AF \perp CB$ and BD is Median.

First draw ΔABC in a Graphical sheet, assume that the point A is an origin. i.e A (0,0).

Since $AB \perp AC$, ($\therefore AC$ is lie on $X - Axis$ & AB is lie on $Y - Axis$)

Hence co-ordinates of B & C are $B(0,2\sqrt{3})$ and $C(2,0)$ respectively.

Now, we find the equation of the line BC, $(x_1, y_1) = B(0,2\sqrt{3})$; $(x_2, y_2) = C(2,0)$

Eqn. of line BC is $\Rightarrow \quad \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$

$$\Rightarrow \quad \frac{x-0}{2-0} = \frac{y-2\sqrt{3}}{0-2\sqrt{3}}$$

$$\Rightarrow \quad -\sqrt{3}\,x = y - 2\sqrt{3}$$

$$\Rightarrow \quad y = -\sqrt{3}\,x + 2\sqrt{3}).$$

This is the equation of the line BC.

Since, $AF \perp CB$, Next we find the equation of the AF,

Since Slope of BC is, $m = -\sqrt{3}$ $\quad (\because y = -\sqrt{3}\,x + 2\sqrt{3})$.

Thus slope of AF is $= \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

Equation of AF is, $y - y_1 = m\,(x - x_1)$ at A (0,0).

$\Rightarrow \quad y - 0 = \frac{1}{\sqrt{3}}(x - 0)$

$\Rightarrow \quad y = \frac{x}{\sqrt{3}}$ ---------------------------- 1

This is the equation of the line AF.

Now, we find the equation of the line BD. $(x_1, y_1) = B(0, 2\sqrt{3})$; $(x_2, y_2) = D(1,0)$

Eqn. of line BD is $\Rightarrow \quad \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

$\Rightarrow \quad \frac{x-0}{1-0} = \frac{y-2\sqrt{3}}{0-2\sqrt{3}}$

$\Rightarrow \quad -2\sqrt{3}\,x = y - 2\sqrt{3}$

$\Rightarrow \quad y = -2\sqrt{3}\,x + 2\sqrt{3}$ ----------------- 2

This is the equation of the line BD.

Since the lines AF and BD are meets at $E(x, y)$.

Solving the equations 1 & 2,

$\Rightarrow \quad \frac{x}{\sqrt{3}} = -2\sqrt{3}\,x + 2\sqrt{3}$

$\Rightarrow \quad x = -6x + 6$

$\Rightarrow \quad x = \frac{6}{7}$

Substitute the value of x in the equation 1, we get $y = \frac{x}{\sqrt{3}}$

$\Rightarrow \quad y = \frac{2\sqrt{3}}{7}$

Hence co-ordinate of E is $\left(\frac{6}{7}, \frac{2\sqrt{3}}{7}\right)$

Now, we find the distance of the line segment AE. $(x_1, y_1) = A(0,0)$; $(x_2, y_2) = E\left(\frac{6}{7}, \frac{2\sqrt{3}}{7}\right)$

$$AE = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$

$$= \sqrt{\left(\frac{6}{7} - 0\right)^2 - \left(\frac{2\sqrt{3}}{7} - 0\right)^2}$$

$$= \sqrt{\left(\frac{6}{7}\right)^2 - \left(\frac{2\sqrt{3}}{7}\right)^2}$$

$$= \frac{1}{7}\sqrt{36 + 12}$$

$$= \frac{1}{7}\sqrt{36 + 12} \quad = \frac{1}{7}\sqrt{48}$$

Hence $AE = \frac{4\sqrt{3}}{7}$