

Solution:

In $\triangle ABC$, $AB = AC$ (given isosceles $\triangle le$)

EFDL, KDIH are square with area 256, 49

Let $BL = y$, $IC = x$, $FD = 16$, $DI = 7$

Let $\angle LBG = \theta = \angle IHC$ ($\because \triangle BKH \sim \triangle HIC$)

$$\text{In } \triangle HIC \tan \theta = \frac{x}{7} \quad \dots \dots \dots (1)$$

$$\text{In } \triangle BKH \tan \theta = \frac{7}{y+9} \quad \dots \dots \dots (2)$$

$$\text{From (1) \& (2)} \frac{x}{7} = \frac{7}{y+9}$$

$$y+9 = \frac{49}{x}$$

$$y = \frac{49-9x}{x} \quad \dots \dots \dots (\text{A})$$

In $\triangle BDC$ as $\angle B = \theta$ $\angle C = 90 - \theta$ $\dots \dots \dots (3)$

And $\triangle ABC \angle C = \frac{180-A}{2} \quad \dots \dots \dots (4)$ (as isosceles triangle)

From (3) & (4) $90 - \theta = 90 - \frac{A}{2}$

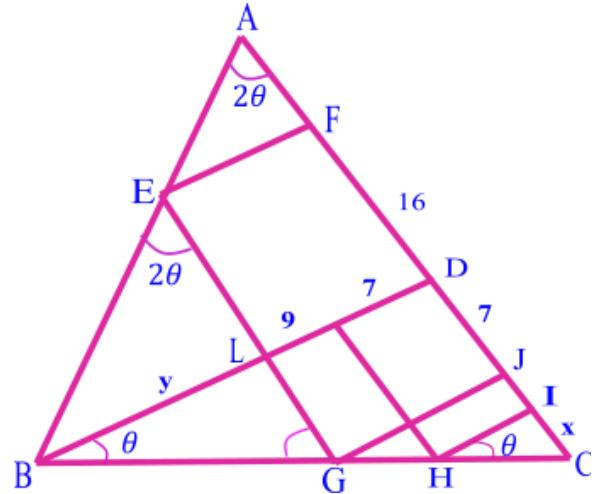
$$\therefore A = 2\theta$$

$$\text{In } \triangle HIC \tan \theta = \frac{x}{7} \quad \dots \dots \dots (5)$$

$$\text{In } \triangle BLE \tan 2\theta = \frac{y}{16} \quad \dots \dots \dots (6)$$

$$\text{W.K.T. } \tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2\frac{x}{7}}{1 - \frac{x^2}{49}} = \frac{2x \times 49}{7(49-x^2)} = \frac{14x}{49-x^2} \quad \dots \dots \dots (7)$$



From (6) & (7)

$$\frac{y}{16} = \frac{14x}{49-x^2}$$

$$\text{By A } \frac{49-9x}{16x} = \frac{14x}{49-x^2}$$

$$(49 - 9x)(49 - x^2) = 14x \times 16x$$

$$2401 - 441x + 49x^2 + 9x^3 = 224x^2$$

$$9x^3 - 273x^2 - 441x + 2401 = 0$$

$$\text{Let } f(x) = 9x^3 - 273x^2 - 441x + 2401$$

By trial method when $x = \frac{7}{3}$, $f(x) = 0$

$$\begin{array}{r|rrrr} \frac{7}{3} & 9 & -273 & -441 & 2401 \\ \hline & 0 & 21 & -588 & 2401 \\ & 9 & -252 & -1029 & \boxed{0} \end{array}$$

$$\therefore 9x^2 - 252x - 1029 = 0$$

$$\text{Solving for } x \quad x = \frac{252 \pm \sqrt{100548}}{18} = \frac{21}{9} [6 \pm \sqrt{57}]$$

The roots are $\frac{7}{3}, \frac{7}{3} [6 \pm \sqrt{57}]$

When $x = \frac{7}{3}$ sub in A

$$x = \frac{\frac{49-9 \times \frac{7}{3}}{7}}{\frac{3}{3}} = \frac{(49-21)^3}{7} = \frac{28}{7} \times 3 = 12$$

$$x = \frac{7}{3}, y = 12$$

$$\text{Required Area} = \frac{1}{2} \times 16 \times 12 + \frac{1}{2} \times 7 \times 21$$

$$= 96 + 73.5 = 169.5 \text{ sq.units}$$

