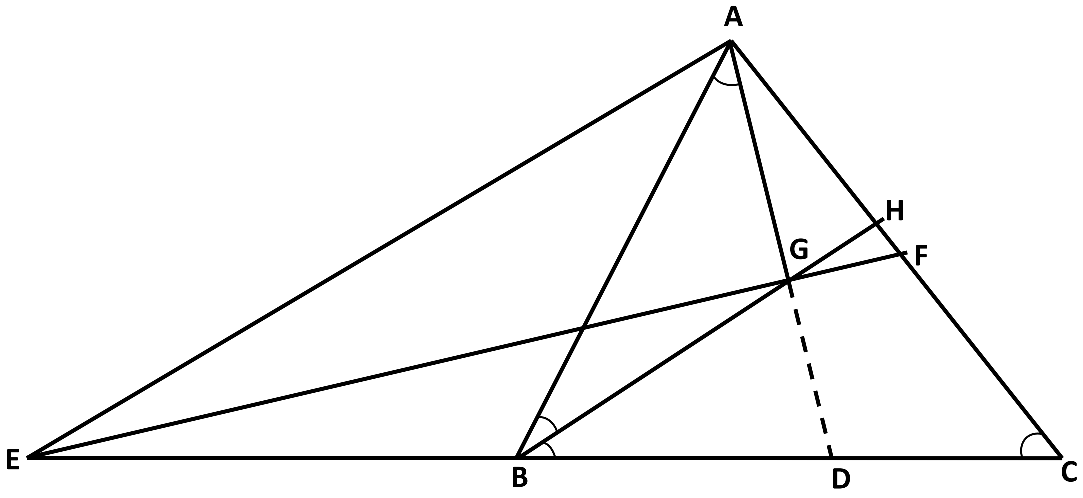
Given:

In Δ ABC, BH is the bisector of $\angle$B meeting AC at H. CB is produced to E such that AB =BE. F is the midpoint of AC. EF and BH meet at G and AG is joined.

To prove: **AG = AH**



Proof:

Produce AG to meet BC at D.

Now, as per Angle Bisector theorem.

$$\frac{AG}{GD}=\frac{AB}{BD} \quad \text{--} (1)$$

For ΔADC, FGE is a transversal.

As per Menelaus Theorem,

$$\therefore \frac{CF}{FA}\times\frac{AG}{GD}\times\frac{DE}{EC}=1$$

Since $\frac{CF}{FA}=1$,

$$\frac{AG}{GD}=\frac{EC}{DE}$$

ie $\frac{AG}{GD}=\frac{AB+BC}{AB+BD}$ (Because AB=EB) ------------------------------ (2)

(1) & (2) $\longrightarrow$

$$\frac{AB}{BD}=\frac{AB+BC}{AB+BD}$$

Since, each ratio is $= \frac{difference \ \ between \ \ numerators}{difference \ \ between \ \ deno\min ators}$

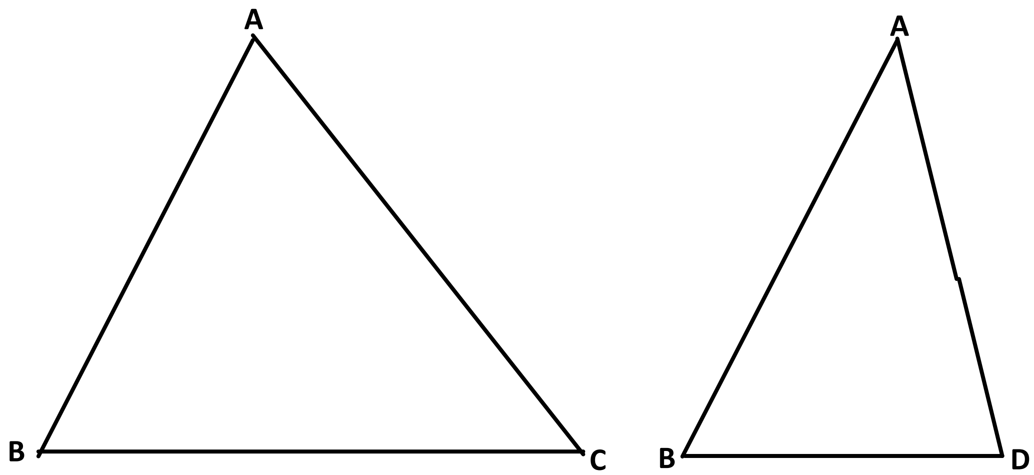
$$\frac{AB}{BD} = \frac{AB + BC - AB}{AB + BD - BD}$$

$$= \frac{BC}{AB}$$

$$\frac{AB}{BD} = \frac{BC}{AB} \text{---------} \ (3)$$

Now, consider the following two triangles

Δ ABC and Δ DBA



$\angle ABD = \angle ABC$

$(3) \longrightarrow \quad \frac{AB}{BD} = \frac{BC}{AB}$

In two triangles, if the ratio between two corresponding sides are equal and the angles those two sides are also equal, then those two triangles are similar. (SAS principle). Therefore those two triangles are similar.

$\therefore$ These triangles are similar.

$\angle BAG = \angle C$ -------------------- (4)

Now, $\angle AHB = \angle C + \angle(\frac{B}{2})$ -------------------- (5)

$\angle AGH = \angle BAG + \angle(\frac{B}{2})$ ------------------(6)

(4), (5) &(6) $\longrightarrow$ $\angle AGH = \angle AHB$

Therefore, AG = AH Proved

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